2020
MATHEMATICS
[HONOURS]
Paper : IV
[SUPPLEMENTARY]

Full Marks : 100 Time : 4 Hours

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Symbols have their usual meanings.

GROUP–A

(Linear Programming and Game Theory)

(Marks : 40)

1. Answer any **two** questions: 1×2=2

   a) Is the optimal solution of an assignment problem unique?

   b) What do you mean by fair game?

   c) Write down the mathematical form of a general L.P.P.

   d) Give an example of a convex set whose all boundary points are extreme points.

2. Answer any **two** questions: 2×2=4

   a) Find, if possible, a basic solution with x_2, a non-basic variable

$$2x_1 - 3x_2 + 5x_3 = 10$$

$$4x_1 + x_2 + 10x_3 = 20$$

   b) Show that whatever may be the value of a, the game with the following payoff matrix is strictly determinable:

		B: I	B: II
A	I	3	7
	II	–3	a

   c) What are the restrictions required to adopt in a travelling salesman problem?

   d) In solving a Transportation problem what is the utility of constructing loops in a transportation table?

3. Answer any **four** questions: 6×4=24

   a) Find the value of the game and the optimal strategies for each player of the game whose payoff matrix is

		B	
	1	−1	−1
A	−1	−1	3
	−1	2	−1

b) Solve the following L.P.P. by Big M-method:

Minimize $Z = 2x_1 + 9x_2 + x_3$

subject to $x_1 + 4x_2 + 2x_3 \geq 5$,

$3x_1 + x_2 + 2x_3 \geq 4$

and $x_1, x_2, x_3 \geq 0$

c) Solve the travelling salesman problem:

		TO				
		1	2	3	4	5
FROM	1	∞	14	10	24	41
	2	6	∞	10	12	10
	3	7	13	∞	8	15
	4	11	14	30	∞	17
	5	6	8	12	16	∞

d) Solve the following transportation problem:

	D_1	D_2	D_3	D_4	a_i
O_1	15	28	13	21	18
O_2	22	15	19	14	14
O_3	16	12	14	31	13
O_4	24	23	15	30	20
b_j	16	15	10	24	

e) Prove that, if any variable of the primal problem unrestricted in sign, then the corresponding constraint of the dual will be an equality.

f) Show that the feasible solution $x_1 = 1$, $x_2 = 1$, $x_3 = 0$ and $x_4 = 2$ to the system:

$$x_1 + x_2 + x_3 = 2$$

$$x_1 + x_2 - 3x_3 = 2$$

$$2x_1 + 4x_2 + 3x_3 - x_4 = 4$$

$$x_1, x_2, x_3, x_4 \geq 0$$

is not basic.

4. Answer any **one** question: 10×1=10

a) i) Prove that, the Transportation problem always has a feasible solution.

ii) Find the optimal assignments for the following assignment problem:

	I	II	III	IV
A	5	3	1	8
B	7	9	2	6
C	6	4	5	7
D	5	7	7	6

4+6=10

b) i) Prove that, the set of all feasible solutions of a linear programming problem is a convex set.

ii) Use duality to solve the L.P.P.:

Minimize $Z = 3x_1 + x_2$

subject to $2x_1 + 3x_2 \geq 2$

$x_1 + x_2 \geq 1$

$x_1, x_2 \geq 0$ 5+5=10

GROUP–B

(Dynamics of a Particle)

(Marks : 50)

5. Answer any **two** questions: 1×2=2

a) State principle of conservation of linear momentum.

b) What is angle of deflection?

c) What do you mean by Impulse?

d) Define time average velocity of a particle.

6. Answer any **five** questions: 2×5=10

a) If a tangential and normal acceleration of a particle moving in a plane curve are equal, find the expression for the velocity.

b) Prove that a planet has only a radial acceleration towards the Sun.

c) A particle describes a curve $r = a e^{\theta}$ with constant angular velocity. Show that its transverse acceleration varies as the distance from the pole.

d) A particle describes the parabola $p^2 = a r$ under a force which is always directed towards its focus. Find the law of force.

e) When the velocity of projection is kept the same, show that there are two different angles of projection for the same range of a projectile.

f) A stone is dropped from a certain height and is observed to fall the last h cm. in t secs. Show that the total time of fall is $\left(\frac{t}{2}+\frac{h}{gt}\right)$ seconds.

g) A particle moves along a straight line according to the following law: $s^2 = 6t^2 + 4t + 3$. Prove the acceleration varies as $\frac{1}{s^3}$.

7. Answer any **three** questions: $6\times3=18$

a) The horse-power required to a steamer of mass M tons at its maximum speed V ft/sec. is H. The resistance is proportional to the square of the speed and the engine exerts a constant propeller thrust at all speeds. If in time t from rest, the steamer acquires a velocity of v ft/sec., prove that $t = \frac{112}{55}\frac{MV^2}{Hg}\log_e\frac{V+v}{V-v}$.

b) A particle describes a parabola under a force which is always directed perpendicularly towards its axis. Show that the force is inversely proportional to the cube of the ordinate.

c) Find the expressions for the radial and cross-radial components velocity and acceleration of a particle moving along a plane curve in polar co-ordinates.

d) A particle of mass m on a straight line is attracted towards the origin on it with a force $m\mu$ times the distance from it and the resistance to motion at any point is mk times the square of the velocity there. If it starts from rest at a distance 'a' from the origin, prove that it will come to rest again at a distance b from the origin, where $(1+2ak)e^{-2ak} = (1-2bk)e^{2bk}$.

e) A particle of mass m moves in a straight line under an attractive force mn^2x towards a fixed point on the line, when at a distance x from it. It is projected with a velocity v towards the centre of force from the initial distance 'a' from it; prove that it reaches the centre of force in time $\frac{1}{n}\tan^{-1}\left(\frac{na}{v}\right)$.

8. Answer any **two** questions: 10×2=20

a) i) A particle is allowed to slide down a smooth inclined plane under gravity alone. Show that the sum of its kinetic and potential energies is always constant throughout its motion.

ii) A particle is projected with velocity V from the cusp of a smooth inverted cycloid down the arc; show that the time of reaching the vertex is $2\sqrt{\frac{a}{g}}\tan^{-1}\left[\sqrt{\frac{4ag}{V}}\right]$. 5+5=10

b) i) A particle moves with a central acceleration $\frac{\mu}{(\text{distance})^2}$; it is projected with velocity V at a distance R. Show that its path is a rectangular hyperbola if the angle of projection is

$$\sin^{-1}\left[\mu\Big/\left\{VR\left(V^2-\frac{2\mu}{R}\right)^{\frac{1}{2}}\right\}\right].$$

ii) A particle moves with a central acceleration $\mu\left(r+\frac{a^4}{r^3}\right)$, being projected from an apse at a distance a with a velocity $2\sqrt{\mu}\,a$; prove that its path is $r^2\left(2+\cos\sqrt{3}\,\theta\right)=3a^2$. 5+5=10

c) i) Two perfectly inelastic bodies of masses m_1 and m_2 moving with velocities u_1 and u_2 in the same direction impinge directly. Show that the loss of kinetic energy due to impact is $\frac{1}{2}.\frac{m_1m_2}{m_1+m_2}(u_1-u_2)^2$.

ii) A gun of total mass M tons, free to recoil horizontally, fires a shot of mass m tons. If the gun is fired with the barrel inclined at an angle α to the horizontal, prove that the shot is actually projected at an angle $\tan^{-1}\left[\left(1+\frac{m}{M}\right)\tan\alpha\right]$ to the horizontal. 5+5=10

GROUP–C

(Analysis–II)

(Marks : 10)

9. Answer any **two** questions: $5\times2=10$

a) Assuming that the petrol burnt in driving a motor-boat varies as the cube of its speed. Show that the most economical speed when going against a current of $c\ \text{km/hr.}$ is $\frac{3c}{2}\ \text{km/hr.}$

b) Find the dimensions of the right circular cone of minimum volume which can be circumscribed about a sphere of radius $8\ \text{cm}$.

c) Find the maximum and minimum values of the function $\cos x \cos(x-\pi/6)\cos(x+\pi/6)$ where $0 \le x \le \pi$.

d) Evaluate:

i) $\lim_{x\to0}\frac{(1+x)^{\frac{1}{x}}-e}{x}$

ii) $\lim_{x\to0}(1+\sin x)^{\cot x}$

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